

## Math 241 Sample Problems for Exam 1

**Question 1** Don't forget cylindrical and spherical coordinates and quadric surfaces.

**Question 2** Let  $f(x, y) = x \sin(y/x)$ . Find the partial derivatives:  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$ ,  $\frac{\partial^2 f}{\partial y \partial x}$

**Question 3** Find and sketch the domain of the function  $f(x, y) = \frac{5}{\sqrt{10 - 2y^2 - x^2}}$ .

**Question 4** Evaluate the following limit:

$$\lim_{(x,y) \rightarrow (1,2)} \frac{\sqrt{x+y} - \sqrt{3}}{x+y-3}$$

**Question 5** Show that the following limit does not exist as  $(x, y) \rightarrow (0, 0)$  by considering different paths of approach.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y + y^2}{x^4 + y^2}$$

**Question 6** Let  $w = f(u, v)$  be a function whose derivatives of all orders exist. Suppose that  $\frac{\partial^2 f}{\partial u^2}(0, 2) = 0$ ,  $\frac{\partial^2 f}{\partial u^2}(3, 0) = -3$ ,  $\frac{\partial^2 f}{\partial u \partial v}(0, 2) = 2$ ,  $\frac{\partial^2 f}{\partial u \partial v}(3, 0) = 3$ ,  $\frac{\partial^2 f}{\partial v^2}(0, 2) = 1$ ,  $\frac{\partial^2 f}{\partial v^2}(3, 0) = -1$ . If  $u = y + e^{2x}$  and  $v = xy$ , what is the value of  $\frac{\partial^2 w}{\partial y^2}$  evaluated at the point  $(x, y) = (0, 2)$ .

**Question 7** Find the direction in which  $f(x, y) = x^2 + \cos xy$  increases most rapidly at the point  $(1, \pi/2)$ . What is the rate at which  $f$  changes in that direction? What is the equation of the tangent plane at the point  $(1, \pi/2)$ ?

**Question 8** Find the critical points of the function

$$f(x, y) = x^4 - x^2y + \frac{3}{4}y^2 - 2y + 5$$

and determine all relative maximum, relative minimum, and saddle points.

**Question 9** A Lagrange multipliers problem